Analysis of Heat and Mass Transfer in Poiseuille Flow between Two Parallel Plates Subjected to an Inclined Magnetic Field

Anita Mishra*, Dr. Sandeep Tiwari

Dept. of Mathematics & Statistics, Integral University, Lucknow–226026, U.P., India
Dept. of Mathematics & Statistics, Integral University, Lucknow–226026, U.P., India

ABSTRACT

In this paper, the effect of heat and mass transfer of an electrically conducting incompressible fluid which moves in presence of magnetic field between two parallel plates and under constant pressure gradient has been studied .after forming the governing equation of motion and choosing the suitable boundary conditions, the solution has been derived for the velocity of fluid and it has been studied analytically and velocity profile has been represented with respect to magnetic field.. it has been found that the magnetic field is very effective in controlling the velocity of fluid and graphically justifying the result of the fluid motion.

KEYWORDS: Poiseuille flow; Porous plate; Conducting fluid; Pressure gradient; Heat and Mass Transfer; inclined magnetic field; Hartmann number.

INTRODUCTION

The flow of liquid metals, motel iron and ionize gasses etc. is very important in industrial field which is studied in magneto-hydro dynamics and this field concerns the motion of an electrically conducting fluid in the presence of magnetic field .It is well known that an electric current is induced when an electrically conducting fluid moves in a magnetic field. Thus MHD generators, MHD pumps, and electromagnetic flow meter are based on MHD flow. Many persons have worked in this field, but Kuri and Bhadur have concentrated on the effect of magnetic field on poiseuille flow between two parallel plates .In the present work focus has been given on the steady MHD two dimensions Poiseuille flow between two plates which are placed in an inclined magnetic field. The concerned differential equations are formed and solved, after that the graphical representation between the magnetic field and velocity has been made and critical studied has been done to find the effect of magnetic on the velocity profile.

FORMULATION OF THE PROBLEM

Let us consider an electrically conducting, viscous, incompressible fluid move between two infinite parallel plates separated by a distance 2d and both plates are at rest. Such a flow indicates a plane Poiseuille flow with constant pressure gradient. In the channel Heat and Mass transfer. Suppose origin be at the centre of the channel with x-axis parallel to the y-axis and perpendicular to the channel walls respectively. The plates are of perpetual figure of length, all the variables except the fluid pressure are function of y.

The plates saturated at y= -1 to 1.fluid flow is horizontal along the x-axis. If u and v are components of velocity v, for an incompressible fluid. The equation of continuity $\nabla \cdot (u, v) = 0$. Reduce the form $\frac{\partial v}{\partial y} = 0, V = V_o$ =constant which indicates, u=u(y). Now, consider V_o be the characteristics velocity perpendicular to the fluid flow maintain a steady state flow at a constant pressure gradient, this V_o at the lower plate is one which will arrange a steady fluid flow against suction and injection.

The governing differential equations are:

$$V_o \frac{\partial u}{\partial y} = \frac{\sigma}{\rho} B_o{}^2 u - \frac{1}{\rho} P + \nu \frac{\partial^2 u}{\partial y^2} + g\beta\left(\overline{T} - \overline{T}_\infty\right) + g\beta\left(\overline{C} - \overline{C}_\infty\right) = 0 \tag{1}$$

and $-\frac{1}{\rho}\frac{\partial p}{\partial y} = 0$ (2)

Where,
Bo: magnetic field
Vo: characteristic velocity

υ_o: kinematic viscosity

σ: electrical conductivity

ρ= fluid density.

The equation (2) takes the fluid pressure P=P(x).Let us assume pressure gradient is constant i.e: dp/dx=constant=p

$$\frac{d^2u}{dy^2} - \frac{V_o}{v}\frac{du}{dy} - \frac{\sigma}{\rho v}B_o{}^2\left(Sin^2\theta\right)u - \frac{1}{\rho v}P + g\beta\left(\overline{T}-\overline{T}_\infty\right) + g\beta\left(\overline{C}-\overline{C}_\infty\right) = 0$$

$$\frac{d^2u}{dy^2} - \frac{V_o}{v}\frac{du}{dy} - \frac{\sigma}{\mu}B_o{}^2\left(Sin^2\theta\right)u - \frac{1}{\mu}P + g\beta\left(\overline{T}-\overline{T}_\infty\right) + g\beta\left(\overline{C}-\overline{C}_\infty\right) = 0 \tag{3}$$

Equation(3) represents the fluid under the influence of an inclined magnetic field. Differentiating (3) with respect to x.

The equation (3) reform:

$$\frac{d^2u}{dy^2} - \frac{V_o}{v}\frac{du}{dy} - \frac{\sigma}{\mu}B_o{}^2\left(Sin^2\theta\right)u - \frac{1}{\mu}P + g\beta\left(\overline{T}-\overline{T}_\infty\right) + g\beta\left(\overline{C}-\overline{C}_\infty\right) = 0 \tag{4}$$

The two fields can be calculated in the nature at an angle θ for $0 \leq \theta \leq \pi$. With boundary condition u=0 for y = $\pm d$.

Using following non-dimensional variables and parameters:

$$\overline{x} = \frac{x}{d}, \overline{y} = \frac{y}{d}, \overline{P} = \frac{\rho d^2}{v^2}, \overline{u} = \frac{ud}{v}, g = -\frac{p}{\mu}, \lambda = \frac{V_o}{v}d, \lambda < 0(\text{suction}), \lambda > 0(\text{injection}),$$

$$M = M^* Sin\theta \text{ or } M = Ha.Sin\theta, \text{ Where } M^* = Ha = B_o d\frac{\sqrt{\sigma}}{\sqrt{\mu}}. = \theta = \frac{\overline{T}-T_\infty}{T_w - T_\infty}, \overline{T}-T_\infty = \theta\left(\overline{T}_W - \overline{T}_\infty\right)$$

$$G_r = \rho L^2 g\beta\left(\overline{T}_w - \overline{T}_\infty\right)/\mu v, G_c = \rho L^2 g\beta\left(\overline{C}_w - \overline{C}_\infty\right)/\mu v; C = \overline{C}-\overline{C}_\infty/\overline{C}_w - \overline{C}_\infty. \tag{5}$$

Then equation (4) becomes:

$$\frac{d^2u}{dy^2} - \lambda\frac{du}{dy} - M^2 u + \left\{g + G_r\theta + G_c C\right\} = 0 \tag{6}$$

Normalize, boundary conditions are:

u =0   at y = 1 and   u = 0 at y = -1 . $\qquad$ (7)

SOLUTION OF THE PROBLEM

The equation (6) is ordinary differential with constant coefficient. The corresponding boundary condition (7) then becomes:

$$u(y) = C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{\left\{g + G_r\theta + G_c C\right\}}{M^2} \tag{8}$$

Where m_1 and m_2 are the roots of the equation obtained from the equation (6) and constants C_1 & C_2 are computed with boundary condition (7) and are obtained as:

$$C_1 = \frac{C}{M^2}\left[\frac{Sinh(m_2)}{Sinh(m_2 - m_1)}\right]$$

and

$$C_2 = \frac{-C}{M^2}\left[\frac{Sinh(m_1)}{Sinh(m_2 - m_1)}\right] \tag{9}$$

Using the constants values to obtained the solution of the equation (6).

It will be expressed as:

$$u(y) = \frac{C}{M^2}\left[1 + \frac{1}{Sinh(2\phi)}\left\{e^{\frac{\lambda}{2}(y-1)}\{Sinh(\phi(y+1))\} - e^{\frac{\lambda}{2}(y+1)}\{Sinh(\phi(y-1))\}\right\}\right] \tag{10}$$

Where $\phi = \sqrt{\left(\frac{\lambda}{2}\right)^2 + M^2}$, $M = Ha.Sin\theta; Ha(Hartmann)$number.

RESULT AND CONCLUSION

Fig.1 shows the variation of injection velocity of fluid with respect to magnetic field expresed in term of Hartman number. On taking θ=15,λ=2,graphs for y=0.005,0.010,0.020 have been obtained. It is clear that as magnetic field increases, the velocity in each case decreases sharply up to a minimum value and then it increases rapidly but for certain value of magnetic field (=30),dips in curves are obtained in each case. Also in the derived relation of velocity (U), the term of magnetic field present in denominator is also existing with the functions of sine and cosine which produce wavy velocity profile. As the value of y increases, the graph shifts upwards this means that the increasing value of y enhances the amplitude of the velocity because of increasing magnetic force. While fig.2 shows the variation of suction velocity of fluid with respect to magnetic field expressed in terms of Hartman's number for same values of θ and y but in the case of suction λ= -2 and the result obtained shows that as magnetic field increases, the velocity decreases similarly as above . On comparing two velocity profiles, it is found that dips are shifting for higher values of magnetic field from suction velocity to injection velocity which is obvious because of magnetic attraction. Also the velocity in each case of injection starts from a little higher value and also achieves higher values than in case of injection. Thus the magnetic field is quite capable of controlling the velocity of fluid in both cases which finds application in industrial use.
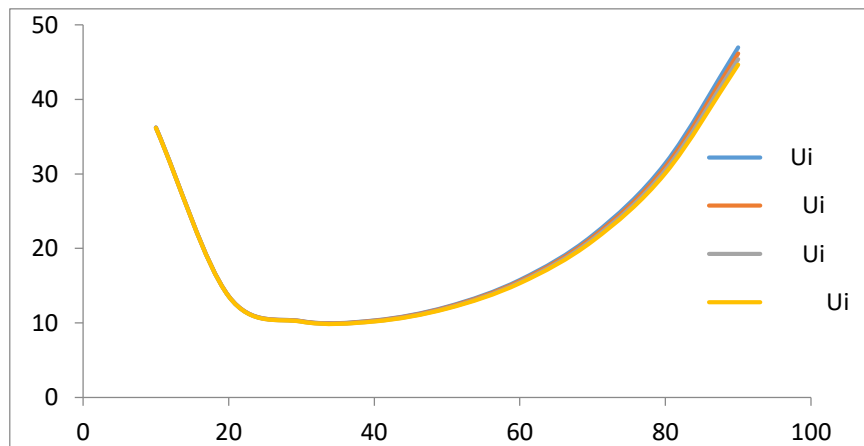

Figure 1: θ=15° , λ =2 g =1.2 case of injection

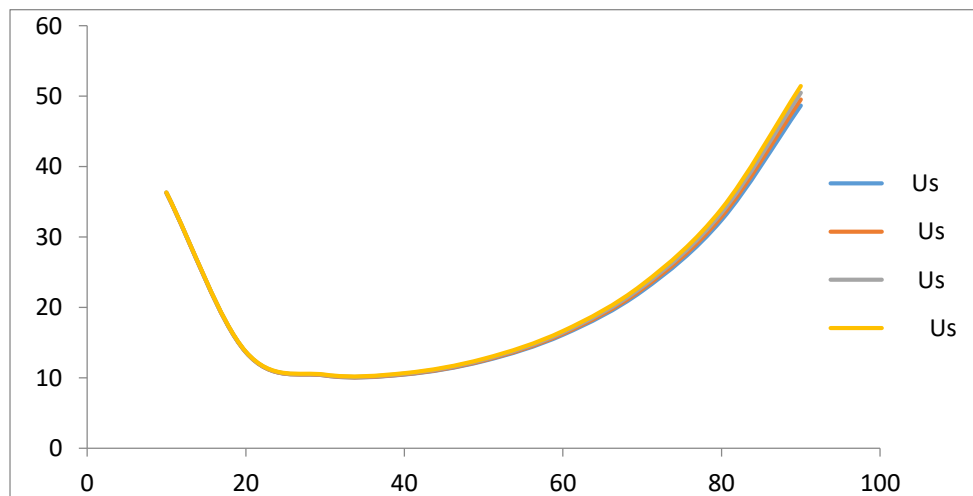*Figure 2: θ=30° , λ = -2 g=1.2 case of suction*

REFERENCES

[1] Das U. N. and Ahmed NFree "Convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall,"Indian Journal Pure Applied Mathematics, Vol. 23, pp. 123-134,(1999).

[2] Dutta N., Dalal D. C. and Mishra S.K.Unsteady heat transfer to palsatile flow of a dusty viscous incompressible fluid in a channel, International Journal of Heat and Mass transfer," Vol. 36, No. 7, pp. 1783-1788,(1993)

[3] Attia H. A.et.al,"Magnetic flow and heat transfer in rectangular channel with variable viscosity. The Arabian Journal for Science and Engineering,"Vol. (30), No. 2A, pp. 1-12,(2005).

[4] Nahme,RBeitrage zur, "hydro-dynamschen theory der lagerreibung,"ing. Arch.,11,191,(1940).

[5] Kuiry.,Bahadur S., "Effect of an inclined magnetic field on an steady poiseuille flow between two parallel plates,"Vol.10,pp.90-96,(2014).

[6] J.S, Waterhouse, and J.G.Kingston, "Plane magnetohydrodynamic flows with constantly inclined magnetic and velocity fields,"J. Appl.Math,Phys.Vol.24,653-658,(1973).

[7] A.Shercliff,"Entry of conducting and non-conducting fluids in pipes ,"J.Math.Proc.of the Cambridge Philos.Soc.52,573-583,(1950).

[8] S. Ganesh and S.Krishnambal, "Unsteady MHDstokes flow of viscous fluid between two   parallel porous plates ,"J . Appl.Sci.7,374-379,(2007).

[9] Bansal,J.L.and Jain.N.C., "Variable viscosity plane Poiseuille flow with unequal wall temprarure,)indian J. Pur Appl.Math,Vol.6(7),800.,(1975).

[10] Alfven,H.,"Existenc of electromagnetic-hydrodynamics waves ,nature,150,(3805),405.(1942).

[11] Ghasemi E., Bayat M. and Bayat M. "MHD flow and heat transfer in a visco-elastic flow over a semi-infinite incompressible and non isothermal stretching sheet."International Journal of Physical Sciences, Vol. 6 (21), pp. 5022-5039, (2011).

[12] Ghosh A. K., Khan A. R. and Debnath L. " On a pulsatile flow of a two-phase viscous fluid is a tube of elliptic cross-section." International Journal of Mathematics and Mathematical Sciences, Vol. (13), No. 4, pp. 669-679(1990).

[13] Mitra P. "Unsteady flow of a dusty viscous incompressible gas through a hexagonal channel," Journal of Engineering and Applied Science, Vol. 2, pp. 277-289,(1983).

[14] Mitra P. and Bhattacharya, P. "The flow of conducting gas between two plates in the influence of transverse magnetic field, Journal of Technology, India, Vol. 26, No.1, pp. 13-24,(1981).

[15] Nag. Kumar S. and Dutta N. "Flow of a dusty fluid through a rectangular channel. Indian Journal of Mathematics, Vol. 27 pp. 377-381,(1989).

[16] Raptis A. et. al ."Unsteady free convective flow through a porous medium, International Journal Engineering Science, Vol. 21, pp. 345-348,(1993).

[17] Reddy Y. B .and et.al. "Unsteady Laminar flow of a fluid with uniform distribution of dust particles through a rectangular channel has been published in Defence Science, Journal, India 22, Vol. 3, pp. 149-156,(1972).

[18] S.Kumar,R.Khare, "Heat and mass transfer in MHDfree convective flow of a visco-elastic (walter's Model-B) dusty fluid through a porous medium,"Vol.5,pp.44-58,(2014). 19.S.Ahsan,R.Khare, "MHD flow of a N0n-Newtonian fluid through an isosceles triangular channel,"Vol.1,Is7,pp94-98,(2015).